THERMAL AND HYDRAULIC CONDITIONS IN A PIPELINE CARRYING A LIQUID

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Nonstationary heat transfer is considered for a pipeline carrying a liquid through cold ground.

Pipeline transport of water or aqueous mixtures or suspensions through cold soils is frequently accompanied by the formation of ice within the pipeline [1-3]. The danger of this is higher in the startup period, when the heat flux to the ground is maximal. Methods are available for calculating the heat transfer from such a liquid to a flat wall [4, 5], to a cold tube [3, 6, 7], and to a low-temperature soil [8], but all these involve the assumption that the temperature of the wall bounding the flow is constant. This corresponds to a steady-state heat condition in the soil and cannot be used for the startup state, where the heat flux to the wall alters very considerably.

Correction for the temperature rise in the soil involves considering the heat conditions in three zones: in the liquid, in the ice layer, and in the surrounding soil.

The problem may be considered as follows. A pipeline of diameter $2a$ is buried in the ground at a depth H from the surface; the temperature distribution in the soil at the start corresponds to the natural temperature distribution in the earth. The pipeline is at the negative natural soil temperature at startup. The pipeline begins to be filled at a constant flow rate Q with liquid at a constant temperature T_0 at the inlet. Then the equation for the heat flow to the liquid is

$$\rho c\left(\pi a^2\frac{\partial T_l}{\partial\tau}+Q\frac{\partial T_l}{\partial z}\right)=-q+N;\quad 0\leqslant z<b; \tag{1}$$

$$\rho c\left(\pi\delta^2\frac{\partial T_l}{\partial\tau}+Q\frac{\partial T_l}{\partial z}\right)=2\pi\delta\alpha(T_p-T_l)+N;\quad z\geqslant b; \tag{2}$$

$$T_l|_{z=0}=T_0. \tag{3}$$

At $z\geq b$ the solid phase (ice) appears in the region $\delta\leq r\leq a$, and for this the conduction equation is

$$\frac{\partial T_i}{\partial\tau}=\varkappa_i\left(\frac{\partial^2 T_i}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial T_i}{\partial r}\right);\quad \delta\leqslant r\leqslant a; \tag{4}$$

$$T_i|_{r=\delta}=T_p; \tag{5}$$

$$l\rho_i\frac{\partial\delta}{\partial\tau}=\alpha(T_l-T_p)+\lambda_i\left.\frac{\partial T_i}{\partial r}\right|_{r=\delta}; \tag{6}$$

$$\delta|_{\tau=\tau_0(z)}=a; \tag{7}$$

$$T_i|_{r=a}=T|_{r=a}. \tag{8}$$

The conduction equation for the soil for $z<b$ is

$$\frac{\partial T}{\partial\tau}=\varkappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right);\quad x^2+(y-H)^2\geqslant a^2;\quad y\geqslant 0; \tag{9}$$

$$T|_{r=a}=T_l; \tag{10}$$

$$T|_{y=0}=T_s; \tag{11}$$

$$T|_{\tau=0}=T_g^0. \tag{12}$$

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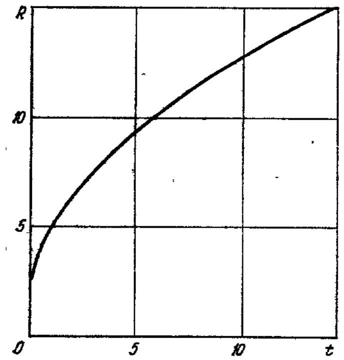

Fig. 1. Dimensionless thermal-influence radius as a function of dimensionless time.

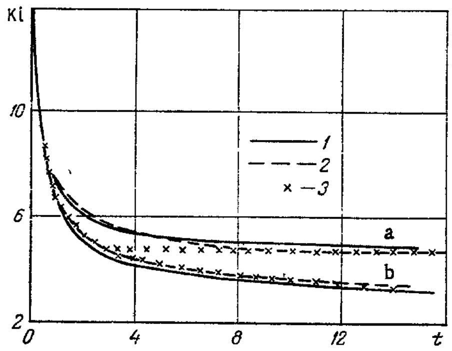

Fig. 2. Comparison of the approximate analytical solution of (18) and (18b) with numerical solution: 1) numerical solution; 2) approximate solution of (18); 3) approximate solution of (18b); a) H = 1 m; b) H = 2.5 m.

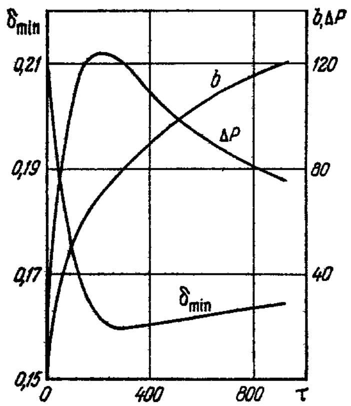

Fig. 3. Time course of the minimal radius δ_{min} of the inner surface of the ice layer, position of the front edge of the ice b, and pressure drop ΔP along the pipeline for $T_n = -6°C$; δ_{min}, m; b, km; ΔP, tech. atm; τ, h.

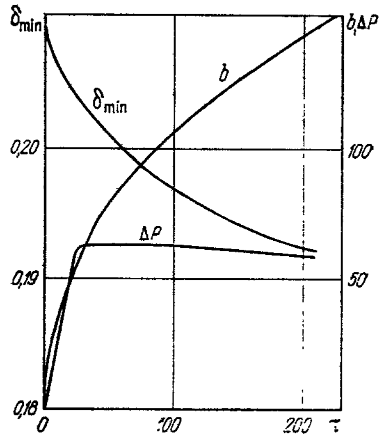

Fig. 4. Time course of δ_{min}, b, and ΔP for $T_n = -2.6°C$.

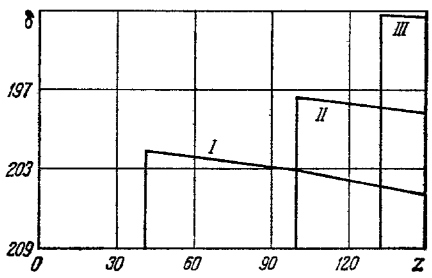

Fig. 5. Shapes of the ice layer at various instants: I) $\tau = 41.1$ h; II) 83.1; III) 163.3; δ, m; z, km.

The boundary condition of (10) has to be replaced by (8) in the zone $z > b$; if it is necessary to allow for the thermal resistance at the wall of the tube and the surface of the ground, one can insert a fictitious additional layer, in which case the boundary conditions of the third kind are reduced to those of the first kind [9, 10].

We assume that the flow of the liquid is turbulent, $3 \cdot 10^3 \leq Re \leq 3 \cdot 10^6$, and then the expression for the hydraulic inclination is [11]

$$i = \frac{Q^2}{(1.8 \lg Re - 1.5)^2 \, 4\pi^2 g R_i^5}. \tag{13}$$

Here

$$R_i = \begin{cases} a & \text{for} \quad z < b \\ \delta & \text{for} \quad z \gg b \end{cases}; \quad \mathrm{Re} = \frac{2Q}{\pi R_i \nu}.$$

There is an internal source of heat N associated with the viscous friction, which is expressed in terms of the hydraulic inclination as follows:

$$N = \rho g Q i / E. \tag{14}$$

System (1)-(14) describes the situation completely; extremely complicated computer programs are required to solve this in general form, and the run times would be very long. The initial system is simplified by applying approximate methods to the parts.

We start with the heat transfer for the soil. We introduce the function $\theta = T - T_g^0$, and we make the obvious assumptions that T_g^0 represents the natural temperature of the ground, which is linear and becomes equal to T_s for $y = 0$, which gives us the following boundary and initial conditions for θ:

$$\theta|_{y=0} = 0; \tag{15}$$

$$\theta|_{r=a} = T_w - T_n = T_b; \tag{16}$$

$$\theta|_{\tau=0} = 0, \tag{17}$$

where T_w is the wall temperature of the tube and T_n is the natural temperature of the ground at the depth of the pipeline. An approximate solution subject to the conditions of (15)-(17) is obtained by supplementing this region with a region $y < 0$; we place an infinite system of fictitious sources on the y axis, which represent a circle of radius a on which the following conditions are met:

$$\theta|_{r_k=a} = A_k = \begin{cases} T_b & k > 0, \\ -T_b & k < 0. \end{cases}$$

The sources $k > 0$ lie on the positive part of the 0y axis, while those with $k < 0$ lie on the negative part. The actual pipeline corresponds to $k = 0$. The other sources lie at the following distances from the axis:

$$\xi_k = \begin{cases} 2k(H - a); & k > 0; \\ 2H - 2(k+1)(H - a); & k < 0. \end{cases}$$

We determine the temperature distribution in the band $0 \le y \le H$ as a sum of axially symmetrical fields set up by this source system. The boundary condition of (15) is met, while (16) is met only at the point on the tube closest to the surface of the ground. The temperature distribution in the region $y > H$ is determined without correction for the surface of the ground. We use the integral method of [12] to determine the axially symmetrical fields and get an expression for the total heat flux at the wall of the pipe:

$$q = -\int_0^{2\pi} \lambda a \frac{\partial T}{\partial r}\bigg|_{r=a} d\varphi = 2\pi\lambda T_b G. \tag{18}$$

Here

$$G = \max\left\{ 1/\ln\left(\frac{H}{a} + \sqrt{\left(\frac{H}{a}\right)^2 - 1}\right); \right.$$

$$\left[1 - \frac{1}{\pi(R-1)} \sum_{k=1}^{M} \left(R \operatorname{arctg} \frac{2(1+q_k)}{1 - 2q_k - q_k^2} + (1+q_k) E\left(\frac{\pi}{4}, d_k\right) + \right.\right.$$

$$+ (1-q_k) F\left(\frac{\pi}{4}, d_k\right) - (3+q_k)\left[E\left(\frac{\pi}{2}, b_k\right) - E\left(\frac{\pi}{4}, b_k\right)\right] + \tag{18a}$$

$$\left.\left.\left. + (1+q_k)\left[F\left(\frac{\pi}{2}, b_k\right) - F\left(\frac{\pi}{4}, b_k\right)\right]\right)\right] \Big/ \left(\frac{R \ln R}{R-1} - 1\right)\right\};$$

$$q_k = 2k\left(\frac{H}{a} - 1\right); \quad d_k = \frac{2\sqrt{q_k}}{1 + q_k};$$

$$b_k = \frac{2\sqrt{2 + q_k}}{3 + q_k}; \quad M = \operatorname{ent}\left[\frac{R-1}{2(H/a - 1)}\right];$$

$F(\varphi, k)$ and $E(\varphi, k)$ are elliptic integrals of the first and second kinds, respectively; int(x) is the integer part of x; and R is the dimensionless thermal-influence function, which is the solution of

$$\frac{dR}{dt} = \frac{\ln R + 1/R - 1}{R \ln R (1 + 1/R + 1/R^2)/6 - R/4 + 1/(4R)};$$

$$R|_{t=0} = 1;$$

where t is dimensionless time (Fourier number). The function R(t) is universal in the sense that it does not contain any parameters for a detailed case; Fig. 1 shows the graph. The error of the approximate solution of (18) was estimated by utilizing a numerical solution (Fig. 2). The Kirpichev criterion was applied. The calculations were performed for the following values of the parameters: $a = 0.5$ m; H = 1 m; H = 2.5 m; $\varkappa = 0.002$ m^2/h.

These calculations show that the method gives good accuracy; the deviations from the numerical solution did not exceed 6%. If the relative depth is comparatively large ($H/a > 2.5$), (18) gives good results if one puts

$$G = \max \left\{ \frac{1}{\ln \left[\frac{H}{a} + \sqrt{\left(\frac{H}{a}\right)^2 - 1} \right]};\ \frac{R-1}{R \ln R - R + 1} \right\}. \tag{18b}$$

This simplification does not introduce additional errors into (18) within the startup period (about 500 h); the physical meaning of (18b) is that the heat flux is determined by the axially symmetrical case up to the point where the steady state is reached. If it is necessary to allow for phase transitions in the soil, (18b) can be modified by inserting an expression for the heat flux obtained for the axially symmetrical two-phase case [13].

We integrate the heat-flux equation of (1) with (18) to get

$$T_l = T_n + \frac{B_3}{G(\eta)} + \left[T - T_n - \frac{B_3}{G(\eta)} \right] \exp\left(-\frac{zG(\eta)}{B_2} \right); \tag{19}$$

where

$$B_2 = \rho c Q / 2\pi\lambda;\ B_3 = N/2\pi\lambda; \text{and}\ \eta = t - \frac{z\varkappa}{wa^2}.$$

The heat transfer is very much accentuated when the flow enters the zone where there is an ice layer. Rough calculations show that one can assume that the temperature of the flow in this zone differs little from T_p. Then we can put the left side of (2) as zero, which gives

$$2\pi\delta\alpha(T_l - T_p) = N. \tag{20}$$

Stefan's condition (7) takes the following form when we use the latter expression and (13):

$$l\rho_i \frac{\partial \delta}{\partial \tau} = \frac{\rho Q^3}{8\pi^3 \delta^6 E (1{,}8 \lg \mathrm{Re} - 1{,}5)^2} + \lambda_i \left. \frac{\partial T_i}{\partial r} \right|_{r=\delta}. \tag{21}$$

Further simplification of (21) is provided by making the usual assumptions that the specific heat of the ice layer can be neglected and that the temperature distribution within the ice layer corresponds to the steady-state distribution for the same boundary conditions [14]. Then we have

$$l\rho_i \frac{\partial \delta}{\partial \tau} = \frac{\rho Q^3}{8\pi^3 \delta^6 E (1.8 \lg \mathrm{Re} - 1.5)^2} + \frac{T_n - T_p}{\frac{\delta}{\lambda_i} \ln \frac{a}{\delta} + \frac{\delta}{\lambda G}}. \tag{22}$$

We see from (22) that the rate of accumulation of the ice layer is determined by the function G, which itself is uniquely determined by the time from the start of the thermal perturbation. Since the latter is constant for each transverse liquid layer in the tube (for a constant flow rate), the thickness of the ice layer after passage of the liquid layer does not vary along the length of the tube. We then introduce the function $\zeta(\xi)$, which represents the internal radius of the ice layer remaining after passage of the liquid layer that passes through the initial cross section at time ξ.

Then (22) takes the following form for this function:

$$l\rho_i \frac{d\zeta}{d\xi} = \frac{\rho Q^3}{8\pi^3 \zeta^6 E (1.8 \lg \mathrm{Re} - 1.5)^2} + \frac{(T_n - T_p)}{\left(\frac{\zeta}{\lambda_i} \ln \frac{a}{\zeta} + \frac{\zeta}{\lambda G(\chi)} \right)}. \tag{23}$$

Here $\chi = \xi a^2/\varkappa$ and the initial condition is

$$\zeta|_{\xi=0} = a. \tag{24}$$

The equation of motion for the leading edge of the ice layer is formulated from the heat-balance condition:

$$\frac{db}{d\xi} = \frac{c\rho (T_l - T_p) a^2 w}{l\rho_i (a^2 - \zeta^2) + \dfrac{2(T_p - T_n)}{\left(\dfrac{1}{\lambda_i} \ln \dfrac{2}{1+\dfrac{\zeta}{a}} + \dfrac{1}{\lambda G(\chi)}\right)}}. \tag{25}$$

Here we have incorporated the fact that the heat released on reducing the temperature of the liquid layer at the leading edge from T_l to T_p is consumed in thawing the leading edge of the ice layer and in heating the soil; then from (19) we have

$$T_l = T_n + \frac{B_3}{G(\chi)} + \left(T - T_n - \frac{B_3}{G(\chi)}\right) \exp\left(-\frac{bG(\chi)}{B_2}\right). \tag{26}$$

We integrate (25) using (26) and the initial condition

$$b|_{\xi=0} = 0. \tag{27}$$

Then one can integrate (23) and (25) by a numerical method to determine $\zeta(\xi)$ and $b(\xi)$, and these functions readily yield the pressure loss in the pipeline and the shape of the ice layer as a function of time. The expression for the former is

$$\Delta P = i(a)\, b + \int_A^{\xi} \frac{Q^3 d\xi}{4\pi^3 g \zeta^7 (1.8 \lg \mathrm{Re} - 1.5)^2}; \tag{28}$$

where

$$A = \max\left\{\xi - \frac{L-b}{w};\ 0\right\}.$$

As an example we consider the startup state for a pipeline having L = 150 km, diameter 0.418 m, and burial depth H = 1.5 m. The pipeline carries water with a flow rate of 741 m^3/h, the temperature at the inlet T being +4°C. The dynamic viscosity of water μ is 6.47 kgf/m · h. The thermophysical characteristics of the soil are $\lambda = 1.465$ kcal/m · h · g, $\varkappa = 0.277 \cdot 10^{-2}$ m^2/h, $T_n = -2.6$°C, and $T_s = -6$°C. Figures 3-5 show graphs for $\delta(z, \tau)$, $b(\tau)$, and $\Delta P(\tau)$. The results indicate that the startup is accompanied by temporary formation of an ice layer on the inner surface. This increases the pressure drop along the pipeline considerably. The maximum pressure difference exceeds the corresponding quantity in the absence of ice by 115%. The ice layer subsequently vanishes and the surrounding ground becomes heated, whereupon the pressure drop falls to the nominal value.

Clearly, the maximum pressure difference along the pipeline, and therefore the maximum pressure at the inlet, will arise shortly after startup, namely, when the soil is at its lowest temperature. An appropriate burial depth and appropriate insulation must be used to ensure that the pressure does not exceed the acceptable value.

NOTATION

T_l, liquid temperature, °C; T_i, ice temperature, °C; T, ground temperature, °C; T_s, surface temperature, °C; T_g^0, natural ground temperature, °C; τ, time h; x, y, z, r, spatial coordinates; a, radius of pipeline, m; H, depth of axis, m; Q, flow rate, m^3/h; w, mean speed, m/h; q, heat flux per meter, kcal/m · h; δ, internal radius of ice layer, m; T_p, phase-transition temperature, °C; N, output of internal heat sources, kcal/m · h; $\varkappa_i$, thermal diffusivity of ice, m^2/h; $\varkappa$, thermal diffusivity of ground, m^2/h; λ_i, thermal conductivity of ice, kcal/deg · m · h; λ, thermal conductivity of ground, kcal/m · h · g; α, liquid–wall heat-transfer coefficient, kcal/m^2 · h · g; Re, Reynolds number; i, hydraulic inclination; g, gravitational acceleration, m/h^2; E, mechanical equivalent of heat, J/kcal.

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ELECTRICAL SIMULATION OF THREE-DIMENSIONAL TEMPERATURE FIELDS OF ANISOTROPIC BODIES OF COMPLEX SHAPE

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A procedure is described and results are presented of mathematical modeling of three-dimensional temperature fields on hybrid electrical combination simulators with integrated microcircuits.

The demands for more accurate thermal calculations increase every year. An increase in the reliability and an improvement of the quality of the elements are inseparably linked with optimization with respect to the thermal state in transient and steady-state regimes. These requirements force one to seek new methods and to solve two- and three-dimensional heat-conduction problems for bodies of complex shape with variable time- and temperature-dependent coefficients in both the basic equation and the boundary conditions of the mathematical model. For the most complex problems the only methods for investigating temperature distributions in three-dimensional structures involve numerical solutions on analog computers with a processor in the form of a network or a combination electrical model [1, 2]. These models are a subroutine of specialized hybrid computers permitting complete automation of the solution of field theory problems described by second-order partial differential equations [3].

In microminiature elements of electronic equipment similar to those shown in Fig. 1, three-dimensional temperature fields can be obtained only by mathematical modeling. The introduction of any temperature-measuring device leads to an inadmissible distortion of the temperature field, particularly inside the microelement. The thermal circuit of a hybrid integrated microcircuit is a typical example of a three-dimensional heat-conduction problem for an anisotropic body of complex shape. The schematic diagram does not show the leads. A special study with three-dimensional electrical models [4] showed that under certain conditions the effect of the leads on temperature fields can be neglected.

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